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Static Pressure Rise in Acoustically Driven Cavities

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PARALLEL uniform flow over a solid wall creates a boundary layer that is laminar or turbulent according to the flow parameters (Reynolds number, roughness of the wall, disturbance level in the flow), and its thickness increases monotonically from the leading edge. If the continuity of the wall is interrupted by a cavity (either a rectangular cutout or a cylindrical well) the boundary layer may separate locally upstream of the cavity and reattach downstream of it. Inside the cavity an unsteady internal flow pattern may develop[1,2] causing pressure fluctuations and intensive acoustic radiation from the orifice of the cavity.[3]

In the present study, it was experimentally established that the presence of an acoustic oscillation inside the cavity strongly alters the mean pressure level there, namely, it causes a steady (d.c.) pressure monotonically increasing with the sound amplitude within the cavity. In order to establish the amplitude and frequency dependence of this pressure rise, the cavity was "driven" by a loudspeaker so that pressure fluctuations of known frequency and amplitude were imposed.

Equipment

The experimental configuration consisted of a flat plate 90 cm long, 30 cm wide, and 0.75 cm thick with beveled leading edge placed into the 30×30 cm open working section of an open return wind tunnel (Fig. 1). The plate was equipped with a cylindrical well of $D=2.54$ cm diameter and of $H=11.35$ cm depth located 11.5 cm downstream from the leading edge. The lower end of the cavity was attached to a low-frequency loudspeaker (woofer) and the whole assembly was hermitically sealed in a flexible plastic bag. This arrangement permitted oscillations and prevented a steady flow through the cavity, so that a steady pressure level could be built up in the cavity. When a static pressure rise developed in the cavity, the plastic bag became fully inflated. The freestream velocity was $U_\infty = 3730$ cm/sec. Measurements were taken under three conditions. The nominal boundary-layer thickness δ (defined as $y=\delta$ where $\bar{U}\ (y)=0.99\ U_\infty$) and the displacement thickness

$$\delta_* = \int_0^\infty (1-\bar{U}/U_\infty)\,\mathrm{d}y$$

were determined for all three cases. The first condition was with no roughness on the plate. The second and third conditions were made by an artificially thickened boundary layer. Two different roughness elements were used: one was produced by a thin step, and the other produced by a thick step attached to the plate near the leading edge (Table 1).

The static pressure in the cavity was measured by using an inclined tube liquid manometer. The velocity amplitude of the imposed oscillations was measured in still air ($U_\infty = 0$) by placing a hot-wire anemometer in the plane of the flat plate at the center of the orifice. From the measured velocity amplitude $\hat{V}$, the displacement amplitude $\hat{A} = \hat{V}/2\pi f$ was calculated to characterize the magnitude of the oscillations.

It was assumed that the amplitude of the oscillation in the cavity with flow was essentially the same as with no flow. In other words, it was assumed that the change in the acoustic impedance of the cavity orifice due to the mean flow was considered negligible.

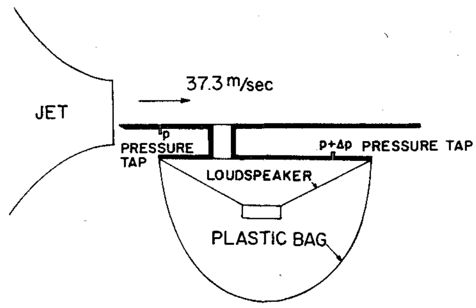

Fig. 1 Experimental facility.

Table 1 Boundary-layer thickness

Roughness (cm)	δ (cm)	δ_* (cm)
No roughness	0.45	0.08
Step 0.127	1.50	0.22
Step 0.208	1.60	0.31

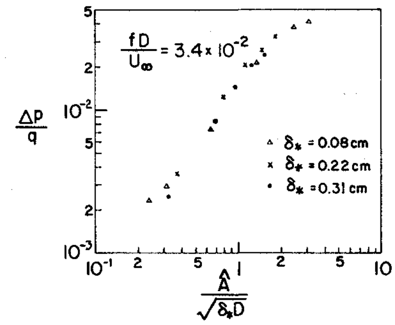

Fig. 2 Data scaling for three different boundary layers,

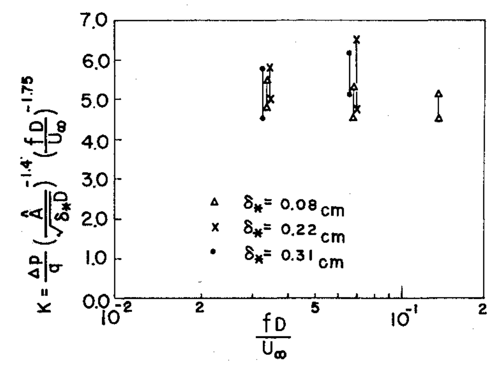

Fig. 3 Determination of constant K.

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Experimental Results

The static (d.c.) pressure in the cavity was measured at all three boundary-layer conditions. It was observed that maximum static pressure developed at a given amplitude with no flow ($U_\infty = 0$) was negligible compared to the pressure developed in the presence of a mean flow. If the loudspeaker was not energized ($\hat{A} = 0$) the pressure rise in the cavity was quite small and reproducible. With both flow and oscillations in the cavity it was found that the pressure rise in the cavity increased monotonically with amplitude $\hat{A}$ and the maximum value reached was about 4% of the dynamic pressure.

Clearly, the set of experiments was of a limited range. Nevertheless, one may speculate about possible scaling laws. The displacement amplitude $\hat{A}$ characterizes the magnitude of the oscillation. The appropriate scaling length must depend on both the boundary-layer thickness $\delta.$ and on the diameter of the well D. The simplest assumption would give both these length scales an equal weight with the choice $\sqrt{\delta.D}$. The previous normalization for a single frequency f resulted in the collapse of all data (Fig.2).

For varying frequencies, it is reasonable to base the normalization on the Strouhal number, defined as $S=fD/U_\infty$. A tentative rough scaling law may be ventured.

$$\Delta p/q = K(\hat{A}/\sqrt{\delta.D})^{1.4}(fD/U_\infty)^{1.75}$$

where $q = 1/2\ \rho\ U_\infty{}^2$ and the nondimensional constant $K=5$. Figure 3 shows that at least the limited data appear to be in fair agreement with the proposed formula.

Discussion

For the explanation of the observed phenomenon the following model is suggested. Due to the oscillation of the air column in the cavity, the boundary layer on the plate is periodically sucked into the cavity and blown out of it. In the suck-in phase a stagnation point is created on the downstream lip of the cavity, and a temporary pressure rise builds up inside the cavity. In the blow-out phase no corresponding negative pressure builds up, so a "rectifier" effect causes the static pressure in the cavity to increase monotonically with the amplitude of the oscillations. At very high amplitudes the pressure increase must reach saturation limit. In many technologically interesting configurations, the static pressure in a cavity may not be assumed equal to the freestream pressure when oscillations are present in the cavity.

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Shock Expansion Analysis of Hypersonic Pistons Decelerating in Long Tubes

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THE existence of light gas guns has made it possible to accelerate light models (0.1 g) to 10,000 km/sec and more massive models to correspondingly lower velocities.[1] Studies of the flow around these high-speed bodies can be made with spark shadowgraph systems, and internal structural damage can be observed with flash x-ray cameras. If the condition of the surface of a model is of interest, the model must be photographed or brought to rest nondestructively. The photography of models requires a high-speed pulsed laser and is difficult. Mechanically stopping the model has been achieved with bales of rags or similar material; however, stopping a model with solid material can drastically alter the model face and obscure experimental details.

The current trend is to attempt to stop a high-speed model by catching it in a tube filled with moderate-pressure gas. The procedure in a typical experiment is to launch the high-speed model with a conventional light gas gun, pass the model through an environment which may disturb its surface (i.e., dust, water drops, etc.), and then to admit the model into a catch tube via a diaphragm section or a fast opening valve.[2] The purpose of the valve is to separate the normally low-pressure test section from a catch tube containing a higher-pressure gas.A simple, long tube at the test section pressure could be used, but would be longer than necessary since most models can stand a higher deceleration than would be generated by maintaining the catch tube at a low pressure.

A shock-expansion analysis is presented here to compute the deceleration of a hypersonic piston in a long tube. If we treat the gas processed by the generated shock as ideal, closed-form solutions are obtained for velocity and position of the piston as a function of time as well as the pressure and temperature at the piston face. The shock wave which runs ahead of the model may be treated as real, but the weakening of the shock due to the model deceleration is ignored. The analytical model is similar to a conventional shock-tube analysis; this treatment differs from the conventional shock-tube analysis in that the model deceleration is caused by the shock pressure, which decays as the model decelerates. Thus, the model dynamics and the gas dynamics are intimately coupled. The following will present an analysis of the decelerating model and the conditions which will occur at its free surface.

Analysis

The affect of the diaphragm rupture or the opening time of a fast valve in the deceleration tube will be presumed to be small. This is justified since stopping distances will usually be large compared to the region disturbed by the opening process.Figure 1 shows the model (or piston) after it has entered the deceleration tube. The piston generates a shock[3] which moves away from it. If the piston velocity were constant, then Fig. 1 would just represent the classical shock-tube problem. In this case the piston is constantly decelerating. As the piston decelerates, expansion waves are generated which weaken the shock, as depicted in Fig. 2. In turn the expansion waves will interact with the shock wave and will generate waves which will return to the piston. Thus, the region between the shock and the piston is filled with a nonsimple wave pattern and must in general be analyzed using the theory of characteristics. Mahoney[4] has shown, for a prescribed piston motion, that shock-expansion theory agrees closely with an exact solution of the theory of characteristics. He found that the entropy gradients produced by the decelerating shock

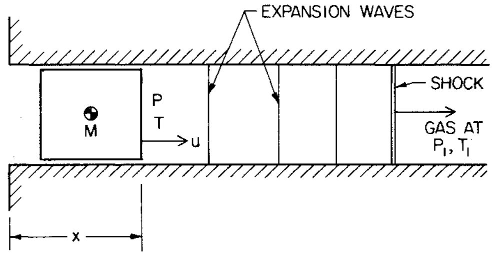

Fig. 1 The model decelerating in a tube. The shock is continuously weakened by expansion moves generated by the piston deceleration.

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